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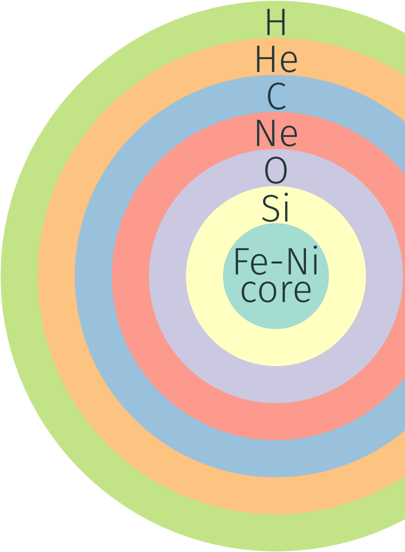
Shell burning

- Stars of initial masses above $\sim 8\mathrm{M}_\odot$
- Shell burning
- Degenerate iron core



Shell burning

Cores must be massive enough to avoid becoming degenerate

- Radiation dominated: $P = \frac{1}{3}aT^4$
- Ideal gas: $P = k\rho T$
- Non-relativistic degenerate electrons: $P = K(\rho/\mu_e)^{\frac{5}{3}}$
- Degenerate electrons: $P = K(\rho/\mu_e)^{\frac{4}{3}}$

Stars in hydrostatic equilibrium:

$$P_c = \beta G(M)^{\frac{2}{3}}(\rho_c)^{\frac{4}{3}}$$ [1]

[1] See lecture notes by Onno Pols for details
https://www.astro.ru.nl/~onnop/education/stev_utrecht_notes/

Shell burning

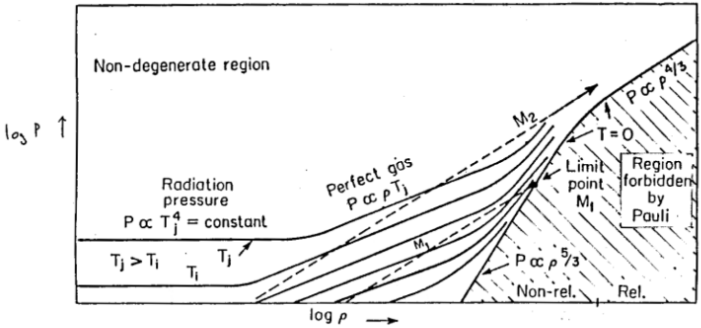
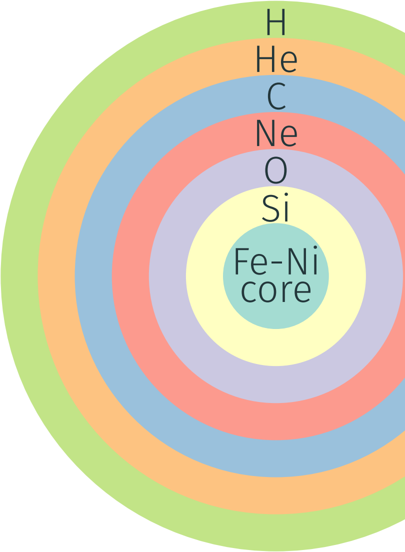


Figure 8.1. Schematic evolution in the $\log \rho - \log P$ plane. Solid lines are isotherms in the equation of state; the dashed lines indicate two evolution tracks of different mass, which have a slope of $\frac{4}{3}$. See the text for an explanation.

Shell burning

- Stars of initial masses above $\sim 8M_{\odot}$
- Shell burning
- Degenerate iron core



Shell burning

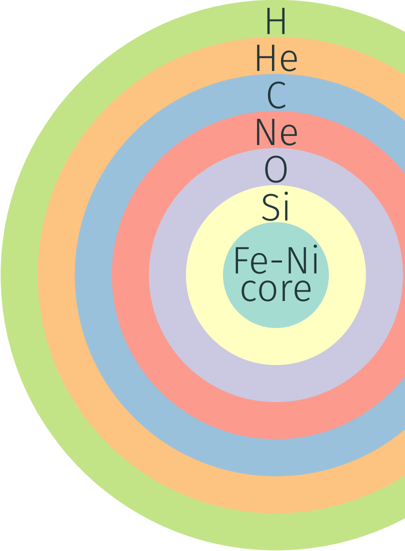
- Short burning periods
- Iron

Table 12.1. Properties of nuclear burning stages in a 15 $M_{\odot}$ star (from Woosley et al. 2002).

burning stage	T (10^9 K)	ρ (g/cm^3)	fuel	main products	timescale
hydrogen	0.035	5.8	H	He	1.1×10^7 yr
helium	0.18	1.4×10^3	He	C, O	2.0×10^6 yr
carbon	0.83	2.4×10^5	C	O, Ne	2.0×10^3 yr
neon	1.6	7.2×10^6	Ne	O, Mg	0.7 yr
oxygen	1.9	6.7×10^6	O, Mg	Si, S	2.6 yr
silicon	3.3	4.3×10^7	Si, S	Fe, Ni	18 d

Shell burning: problems with this picture?

- What about multi-dimensional effects?
- Stellar evolution as a whole
- Winds, magnetic fields, and mass loss



Collapse of the Iron-core

- Chandrasekhar mass limit $\mathrm{M_{Ch}} = 5.8\,\mathrm{Y_e^2}\,\mathrm{M_\odot}$ [1]
- Cold ^{56}Fe white-dwarf, $\mathrm{M_{Ch}} = 1.10618$ [3]

$$\mathrm{M_{Ch}} = 1.44\left(\frac{\mathrm{Y_e}}{0.5}\right)\left[1 + \frac{\mathrm{s_e}}{\pi \mathrm{Y_e}}\right] [4]$$

Collapse

Collapse of the Iron-core

A runaway process

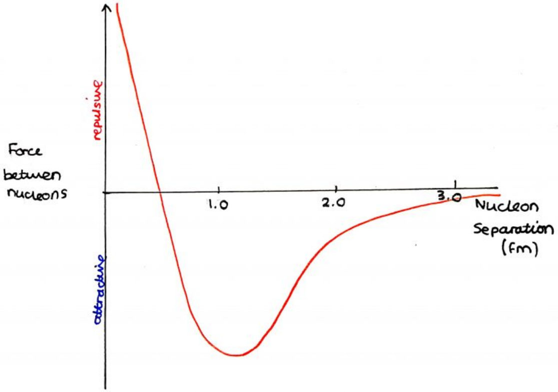
$$^{56}\mathrm{Fe} + \gamma \rightarrow 13\,^{4}\mathrm{He} + 4\mathrm{n}$$

$$\mathrm{p} + \mathrm{e}^{-} \rightarrow \mathrm{n} + \overline{\nu}_{\mathrm{e}}$$

Continues until $\rho \sim 2 \times 10^{14}\,\mathrm{g/cm^3}$, at which point the strong nuclear force kicks in.

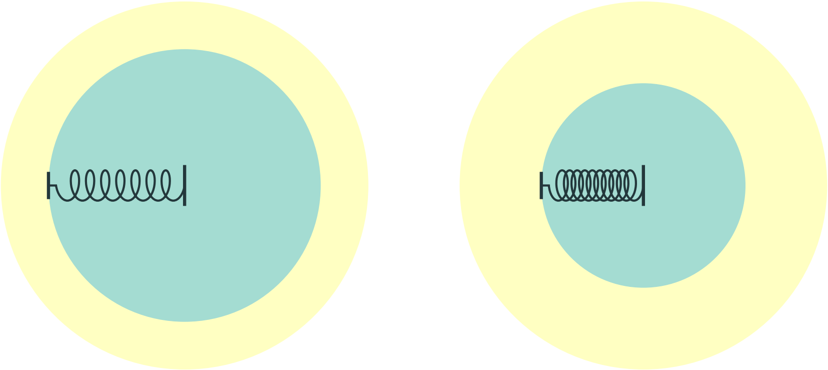
Core Bounce

- Nuclear densities
- Inner core stops contracting



Core Recoil

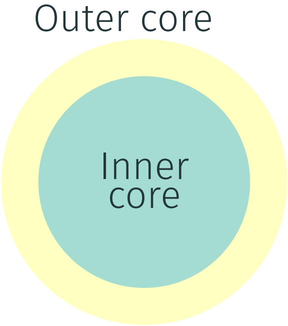
The strong force acts like a spring



What does a compressed spring do?

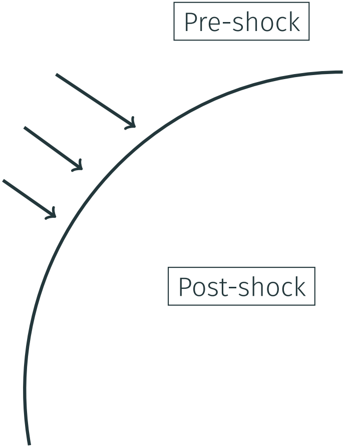
Core Bounce

- Inner core stops contracting
- Supersonically infalling outer core
- Shock wave



Shock Propagation

- Disintegration of heavy nuclei
- Density decreases
- Neutrino burst



Shock Revival

Neutrino Heating

Heating:

$$n + \nu_e \rightarrow p + e^-$$
$$p + \bar{\nu}_e \rightarrow n + e^+$$

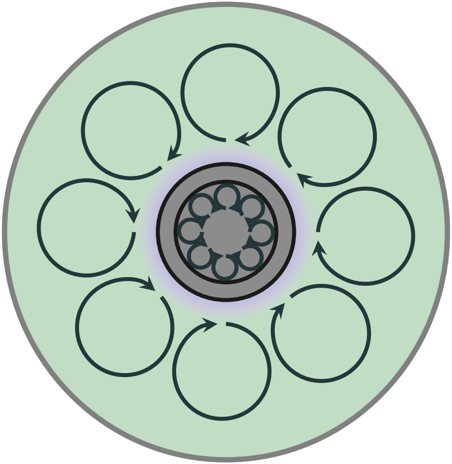
Rate $\sim 1/\mathrm{r}^{-2}$

Cooling:

$$p + e^- \rightarrow n + \nu_e$$
$$n + e^+ \rightarrow p + \bar{\nu}_e$$

Rate $\sim 1/\mathrm{r}^{-6}$

See [5] for a detailed derivation

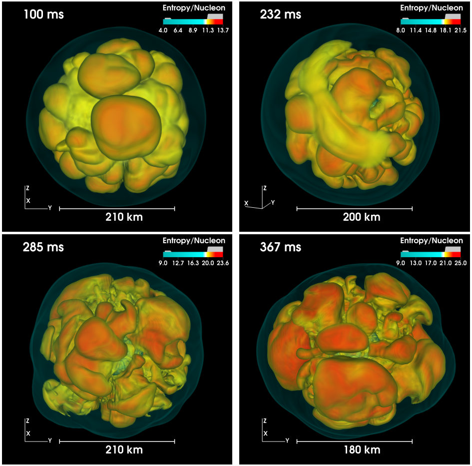


Hydrodynamics

Movie time

Hydrodynamics

- The standing accretion shock instability (SASI)
- Hot bubble convection
- Convection in the interior of the Proto-neutron star (PNS)



Turbulence

- Turbulence is key
- Material spends more time in the heating region
- One, two, and three dimensions

Energy cascade

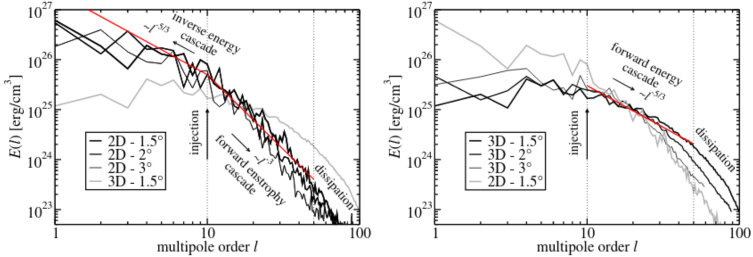


Figure 16. Turbulent energy spectra $E(l)$ as functions of the multipole order l for different angular resolution. The spectra are based on a decomposition of the azimuthal velocity v_θ into spherical harmonics at radius $r = 150$ km and 400 ms post-bounce time for 15 $M_\odot$ runs with an electron–neutrino luminosity of $L_{\nu_e} = 2.2 \times 10^{52}$ erg s^{-1}. Left: 2D models with different angular resolution (black, different thickness) and, for comparison, the 3D model with the highest employed angular resolution (gray). Right: 3D models with different angular resolution and, for comparison, the 2D model with the highest employed angular resolution (gray). The power-law dependence and direction of the energy and enstrophy cascades (see the text) are indicated by red lines and labels for 2D models in the left panel and 3D models in the right panel. The left vertical, dotted line roughly marks the energy-injection scale, and the right vertical, dotted line denotes the onset of dissipation at high l for the best-displayed resolution.

Post Explosion

- Proto-neutron star cools to a neutron star
- Fallback and black holes
- Nucleosynthesis and matter ejection

Numerical Simulations

Input Physics

- Stellar evolution
- Hydrodynamics
- General relativity
- Neutrino transport
- Equation of state
- Neutrino interactions
- Turbulence

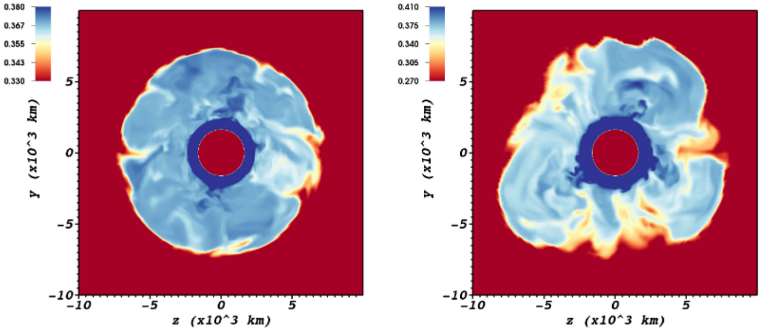


Figure 2. Slices showing the mass fraction X_{Si} of silicon at the onset of collapse in models s18-3Dr (left) and s18-3D (right). Both models are characterized by 2–3 silicon rich plumes (darker shades of blue. Due to the higher convective velocities, the boundary between the oxygen shell and the carbon shell is more strongly distorted by interfacial wave breaking in model s18-3D.)

Müller et. al. 2017 [9]

Stellar Evolution

- Stellar structure
- Rotation
- Magnetic fields
- Mass loss

Hydrodynamics

- Multi-dimensional
- Numerical setup and grid structure
- Turbulence

Turbulence

- Can you ever reach the scales you need?
- Melson et. al. 2020 [8]
- Convergence around an angular resolution of 1 degree

General Relativity

- Adds complexity and cost
- Often a “post-newtonian” approach is used [7]

$$m_{\mathrm{TOV}} = 4\pi \int_0^{r} dr' r'^2 \left[\rho + e + E + \frac{vF}{r}\right]$$

$$\Phi_{\mathrm{TOV}}(r) = 4\pi \int_r^{\infty} \frac{dr'}{r'^2} \left[\frac{m_{\mathrm{TOV}}}{4\pi} + r'^3 \left(P + P_\nu\right)\right] \times \frac{1}{\Gamma} \frac{\rho + e + P}{\rho}$$

$$\Phi = \Phi - \overline{\Phi} + \Phi_{\mathrm{TOV}}$$

Equation of State

- High density and temperature
- Low density regime
- Exotic particles?

Neutrino Interactions and Transport

Boltzmann equation

$$p^\alpha \left[\frac{\partial f_\nu}{\partial x^\alpha} - \Gamma^\beta_{\alpha\gamma} p^\gamma \frac{\partial f_\nu}{\partial p^\beta} \right] = \left[\frac{df_\nu}{d\tau} \right]_{coll}$$

Several approximations exists

- Ray-by-Ray
- Grey transport
- M1 and M2

Neutrino Interactions and Transport

Table 1 Most important neutrino processes in supernova and proto-neutron star matter.

Process	Reaction[a]
Beta-processes (direct URCA processes)	
electron and ν_e absorption by nuclei	$e^- + (A,Z) \longleftrightarrow (A,Z-1) + \nu_e$
electron and ν_e captures by nucleons	$e^- + p \longleftrightarrow n + \nu_e$
positron and $\bar{\nu}_e$ captures by nucleons	$e^+ + n \longleftrightarrow p + \bar{\nu}_e$
"Thermal" pair production and annihilation processes	
Nucleon-nucleon bremsstrahlung	$N + N \longleftrightarrow N + N + \nu + \bar{\nu}$
Electron-position pair process	$e^- + e^+ \longleftrightarrow \nu + \bar{\nu}$
Plasmon pair-neutrino process	$\tilde{\gamma} \longleftrightarrow \nu + \bar{\nu}$
Reactions between neutrinos	
Neutrino-pair annihilation	$\nu_e + \bar{\nu}_e \longleftrightarrow \nu_x + \bar{\nu}_x$
Neutrino scattering	$\nu_x + \{\nu_e, \bar{\nu}_e\} \longleftrightarrow \nu_x + \{\nu_e, \bar{\nu}_e\}$
Scattering processes with medium particles	
Neutrino scattering with nuclei	$\nu + (A,Z) \longleftrightarrow \nu + (A,Z)$
Neutrino scattering with nucleons	$\nu + N \longleftrightarrow \nu + N$
Neutrino scattering with electrons and positrons	$\nu + e^{\pm} \longleftrightarrow \nu + e^{\pm}$

[a] N means nucleons, i.e., either n or p, $\nu \in \{\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau\}$, $\nu_x \in \{\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau\}$

See [6]

Neutrino Interactions and Transport

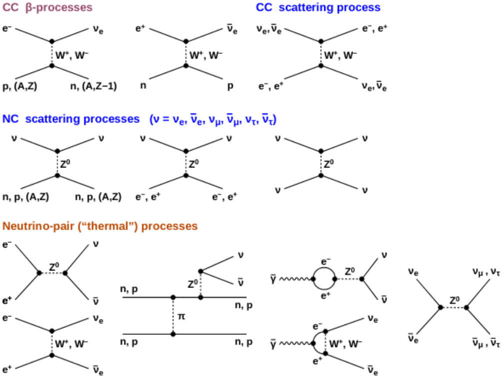


Fig. 3 Feynman diagrams for the lowest-order contributions to the most relevant neutrino interactions in supernova cores. Charged-current (CC) reactions are mediated by $W^{\pm}$ bosons, neutral-current (NC) reactions by electrically neutral Z^0 bosons. The charged-current β-processes are responsible for the production and absorption of ν_e and $\bar{\nu}_e$ by lepton-capture reactions on nucleons (*top row, left*). Scattering processes include the charged-current interactions of ν_e and $\bar{\nu}_e$ with electrons and positrons (*top row, right*) and neutral-current scatterings of neutrinos and antineutrinos of all flavors with nuclei, neutrons, protons, electrons, positrons, and neutrinos (*middle row*). Neutrino-pair processes are responsible for the creation and annihilation of neutrino-antineutrino pairs of all flavors. They include electron-position pair annihilation through neutral and charged currents, nucleon bremsstrahlung, the charged- and neutral-current plasmon-neutrino processes, and neutrino-pair conversion between different flavors (*bottom row, from left to right*).

See [6]

Neutrino Interactions and Transport

TABLE I. Neutrino reactions with muons.

$\nu + \mu^- \rightleftarrows \nu' + \mu^{-\prime}$	$\nu + \mu^+ \rightleftarrows \nu' + \mu^{+\prime}$
$\nu_\mu + e^- \rightleftarrows \nu_e + \mu^-$	$\bar{\nu}_\mu + e^+ \rightleftarrows \bar{\nu}_e + \mu^+$
$\nu_\mu + \bar{\nu}_e + e^- \rightleftarrows \mu^-$	$\bar{\nu}_\mu + \nu_e + e^+ \rightleftarrows \mu^+$
$\bar{\nu}_e + e^- \rightleftarrows \bar{\nu}_\mu + \mu^-$	$\nu_e + e^+ \rightleftarrows \nu_\mu + \mu^+$
$\nu_\mu + n \rightleftarrows p + \mu^-$	$\bar{\nu}_\mu + p \rightleftarrows n + \mu^+$

See [2]

Numerical Simulations

- Many different things goes into the simulations
- Uncertainties in almost every aspect
- Approximations are necessary

Performing Simulations

Numerical Codes

- GR1D: *https://github.com/evanoconnor/GR1D*
- NuLiB: *https://github.com/evanoconnor/NuLib*
- FLASH: *http://flash.uchicago.edu/site/flashcode/*
- Einstein Toolkit: *https://einsteintoolkit.org*
- Specter: *https://github.com/sxs-collaboration/spectre*
- Quokka: *https://github.com/BenWibking/quokka*

Black Hole Formation

- GR1D
- A 75 $M_\odot$ star
- Polytropic equation of state

Setting up GR1D

```
git clone https://github.com/evanoconnor/GR1D.git
cd GR1D
```

Setting up GR1D

```
F90=
F90FLAGS=-g -O3 -fopenmp
LDFLAGS=-g -O3 -fopenmp
MODINC="-I./"

HDF5DIR=
HDF5INCS=-I${HDF5DIR}/include
HDF5LIBS=-L${HDF5DIR}/lib -lhdf5 -lhdf5_fortran -lhdf5 -lz

LAPACKDIR=
LAPACKLIBS=-L${LAPACKDIR}/lib -llapack -lblas

HAVE_NUC_EOS=1
HAVE_RESTART=1
HAVE_LEAK_ROS=0
HAVE_LAPACK=1
```

Setting up GR1D

```
cp sample_parameter_files/latest_recommended_params ./parameters

##### Job parameters #######
jobname = "BHF"
GR = 1 # 1 for GR, 0 for Newtonian
outdir  =  "u70-Black-hole"
initial_data = "Collapse"
profile_name = "./u70.short" # stellar profile
WHW02profile = 1
profile_type = 1
gravity_active = 1 # do we want gravity?
ntmax = 100000000 # maximum timestep
tend = 0.7d0 # maximum time
```

Setting up GR1D

```
####### Grid parameters #######
geometry = 2 # 1: planar, 2: spherical
gridtype = "custom" # "log", "unigrid", "custom", "custom2"
### custom & custom2 input parameters ###
grid_custom_rad1 = 20.0d5
grid_custom_dx1 = 3.0d4 # smallest radial zone

rmax_from_profile = 1 # take rmax to be where rho=3.0d3g/cm^3
rho_cut  = 2.0d3 # density to cut profile at if rmax_from_profile = 1

radial_zones = 600 # number of radial zones
ghosts1 = 4 # number of ghost cells
```

Setting up GR1D

```
####### EOS parameters #######
eoskey = 1 # hybrid: 1 # poly: 2 # hot nuclear: 3 # ideal: 4
eos_table_name  =  ""
hybridgamma_th = 1.30d0 # hybrid gamma_th
hybridgamma1 = 1.31d0 # hybrid gamma_th
hybridgamma2 = 2.40d0 # hybrid gamma_th
```

Setting up GR1D

```
########## M1 settings ##########
do_M1    = 0 #1 for M1 transport scheme

####### Neutrino parameters #######
fake_neutrinos = 0 # 1 for ANY fake neutrino scheme
```

Running GR1D

```
Finding envelope binding energy
Binding energy of envelope (ergs):   3.4481747535627853E+051
Finding accretion radii
Setting up PPM coefficients
Done with initial data :-)
Begin time integration loop:
      0   0.000000E+00   1.050000E-07   1.368096E+09   9.934271E-01
    100   2.740526E-04   1.380763E-05   1.385673E+09   9.934264E-01
    200   2.474790E-03   2.256444E-05   1.383073E+09   9.934198E-01
    300   4.731547E-03   2.253527E-05   1.388112E+09   9.934114E-01
    400   6.980886E-03   2.244709E-05   1.401145E+09   9.934012E-01
    500   9.220404E-03   2.233893E-05   1.422465E+09   9.933892E-01
```

GR1D Outout

```
cd u75-bh
ls

M_innercore.dat      eps_kin.xg                 press.xg
W.xg                 list_of_restartfiles.txt   pressth.xg
X.xg                 mass_bary.xg               r_Xmax.dat
accreted_mass.dat    mass_grav.xg               r_rho1e11.dat
accretion_rates.dat  mbary_Xmax.dat             r_rho1e12.dat
alpha.xg             mbary_rho1e12.dat          ye.xg
alpha_c_t.dat        mbary_shock.dat            ye_c_t.dat
cs.xg                mgrav_Xmax.dat             ynu_c_t.dat
csound_c_t.dat       mgrav_rho1e12.dat          rho.xg
dyedt_hydro_c_t.dat  mgrav_shock.dat            rho_c_t.dat
eps.xg               parameters                 time_c.dat
```

GR1D Output

Different output files

- .xg : Radial profiles at several time-steps
- .dat : Special data versus time
- _c.dat Values at the central zone versus time

The structure of the .xg files depend on the grid and the output frequency can be changed in the parameter file.

Exercise

- Repeat the evolution
- Tabulated EOS
- M1 transport
- Bounce time, black hole formation, difference in the evolution?

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